Forty years entangled in networks.

Eric Goles

¡Bienvenido

Maximilien

Antun

Goles!


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Topics:

1) Neural or Threshold Networks: dynamics and energy

2) Regulation Networks: dynamics and Robustness.

3) Ants models and its complexity.

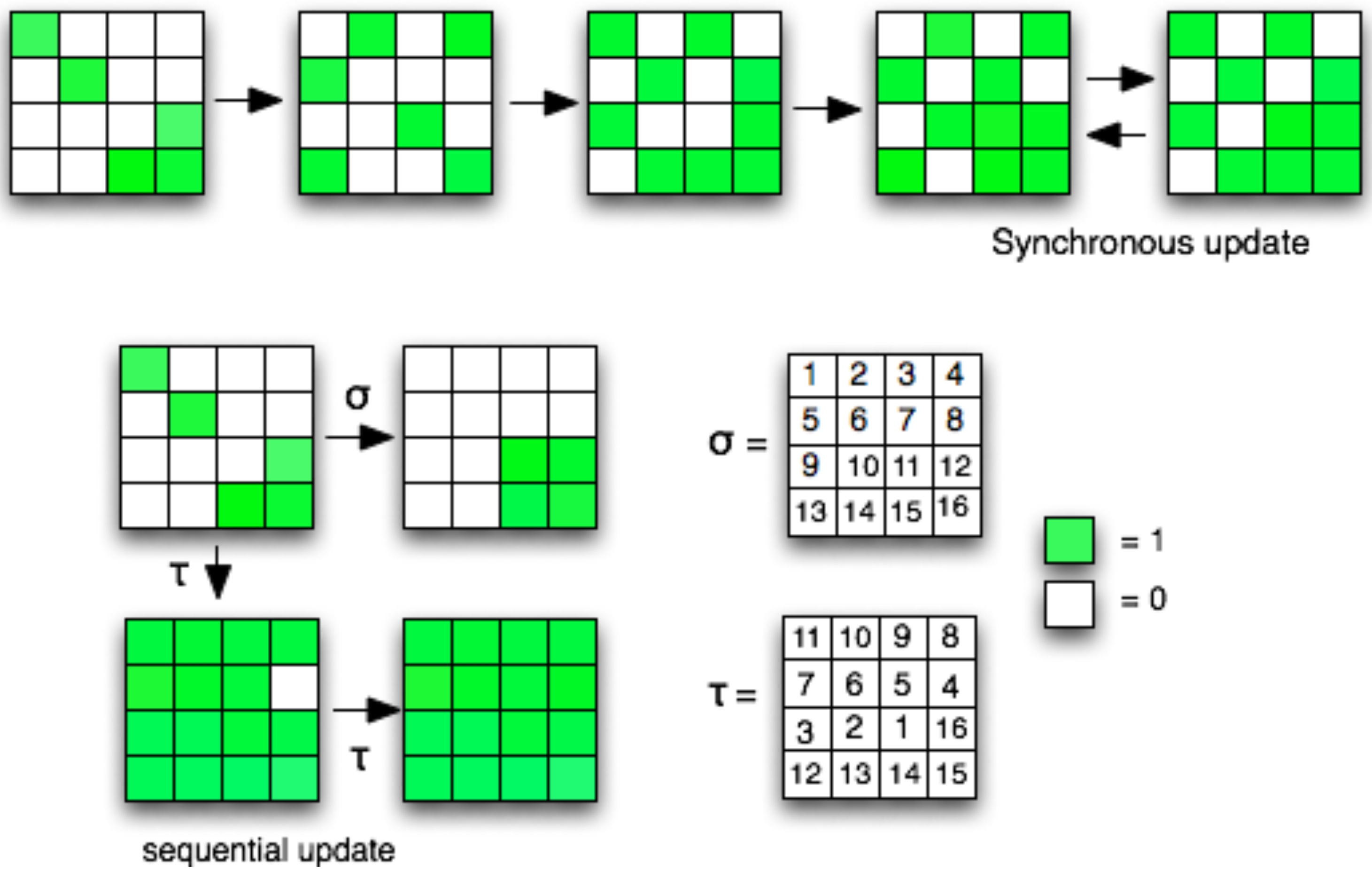
4) Sand Piles: universality and lattice structure.

5) Prediction and Complexity

6) Social Science: Schelling Segregation, Sakoda's model and polarization

NEURAL AND THRESHOLD NETWORKS

$$x'_{ij} = 1 \quad iff \quad x_{i-1,j} + x_{i+1,j} + x_{i,j-1} + x_{i,j+1} \geq 2$$



We consider a 4x4 lattice with periodic conditions, nearest interactions, states 0 or 1, and the local majority function: If the number of ones is bigger or equal to the number of zeros then the site takes the value 1.

Neural networks

$$x'_i = s\left(\sum_{j=1}^{n} w_{ij} x_j - b_i\right) \quad for \quad 1 \leq i \leq n$$

$W = (w_{ij})$ The weight matrix

$b = (b_i)$ The threshold vector

$s(u) = 1$ iff $u \geq 0$

0 otherwise

For arbitrary matrices W previous model may accept, iterated in parallel or block-sequentially, long period cycles and long transients … But when W is symmetric the network admits short periods and an energy: (E.G and J.Olivos, Discrete Mathematics, 1980, Discrete Applied Maths, 1981; E.G, SIAM J of Computing, 1982; E:G, F. Fogelman, Discrete Applied Maths(1985))

$$E(x(t)) = -\sum_{i=1}^{n} x_i(t) \sum_{j=1}^{n} w_{ij} x_j(t-1) + \sum_{i=1}^{n} b_i (x_i(t) + x_i(t-1))$$

Further, if diag (W) ≥ 0, any sequential update admits the energy (E.G., F. Fogelman, G. Weisbuch, Disc. Applied Maths. 1982)

$$E(x) = -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} x_i x_j + \sum_{i=1}^{n} b_i x_i$$

For arbitrary matrices W previous model may accept, Iterated in parallel or sequentially, long period cycles and transients …..

But when W is symmetric the network converges to fixed point or two periodic cycles (parallel update),

And, if diag(W)≥0 to fixed point (sequential update).

E.G, J. Olivos, Periodic behaviour of generalized threshold functions, Discrete mathematics, vol 30, pp 187-189, 1980

E.G., Fixed Point behavior of threshold functions on a finite set, SIAM Journal on Alg. And Discrete Methods, vol 3(4), pp 2554-2558, 1982.

The most general dynamical result:

Consider the block-sequential scheme $s = \{I_1, \ldots, I_p\}$

The symmetrical threshold network T=(W, b, s)

Let $W(I_k)$ the sub-matrix associated to the k-th block

If for every $k \in \{1, \ldots, p\}$ $W(I_k)$ is non-negative-definite

The network converges to fixed points

E. G., F. Fogelman-Soulie, D. Pellegrin, Decreasing energy functions as a tool

For studying threshold networks, Discrete Applied Mathematics, vol 12, pp261-277, 1985.

We will suppose now that every matrix is the incidence matrix of an undirected graph G=(V,E), so their entries belong to the set {0,1}
W=W(G)= (w_{ij}) eventually with loops $(w_{ii}=1)$

Consider the quantity:

$$\alpha(G) = -n - k + 2m - 4p$$

n = |V|,
m =|E|, (without loops)
K = the number of loops,
P = the minimum number of edges to remove such that the sub-graph is bipartite.

Contents lists available at ScienceDirect
Neural Networks
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ELSEVIER

Dynamics of neural networks over undirected graphs

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Theorem: attractors for every block-sequential update.

Consider the block-sequential scheme $s = \{I_1, \ldots, I_p\}$

The symmetrical threshold network *T*=(W, b, *s*)

Let $G(I_k)$ the graph associated to the k-th block

$\forall\, k \in \{1, \ldots, p\}\;\; \alpha(G') < 0 \;\; \forall\, G' \subseteq G(I_k) \;\; \Rightarrow$ fixed points

$\exists\, k \in \{1, \ldots, p\}$ and $G' \subseteq G(I_k)$ such that $\alpha(G') \geq 0 \;\Rightarrow$ cycles

FAUT-IL ADOPTER L'AVIS DE SES VOISINS ?

La modélisation de réseaux d'individus dont l'opinion se conforme à celle de la majorité de leurs relations montre des évolutions parfois inattendues où, par exemple, le réseau se met à osciller entre deux états contradictoires.

L'AUTEUR

JEAN-PAUL DELAHAYE
professeur émérite à l'université de Lille et chercheur au Centre de recherche en informatique, signal et automatique de Lille (Cristal)

Jean-Paul Delahaye a récemment publié : **Les Mathématiciens se plient au jeu**, une sélection de ses chroniques parues dans *Pour la Science* (Belin, 2017).

Dans une communauté ayant une décision à prendre, par exemple quant à la construction d'un pont, un vote doit avoir lieu. Les électeurs sont conciliants : chacun a un avis, OUI ou NON, mais il est prêt à y renoncer si, parmi ses connaissances, plus de la moitié des avis sont contraires au sien. Pendant plusieurs jours, après un tour initial, chacun des électeurs consulte toutes ses connaissances et change d'avis le lendemain sur la construction du pont pour s'ajuster à l'avis majoritaire de ses connaissances consultées la veille. En cas d'égalité entre les pour et les contre parmi ses voisins, l'électeur garde le même avis pour le lendemain. Que va-t-il se passer ?

Ce problème en apparence élémentaire des « dynamiques majoritaires » agite les mathématiciens depuis bientôt 40 ans. Ils en ont découvert une multitude de propriétés inattendues. Le premier résultat important à son sujet a été démontré en 1980 par mon ami Eric Goles, alors doctorant à l'université de Grenoble et aujourd'hui à l'université Adolfo Ibáñez, à Santiago du Chili, et Jorge Olivos, de l'université du Chili : cette dynamique conduit soit à une stabilisation des opinions des électeurs, soit à une oscillation des opinions entre deux configurations C et C' : la configuration C donne le lendemain C' et C' donne le lendemain C. Aucune autre situation (oscillation entre trois états ou plus, désordre prolongé...) n'est possible.

La modélisation du problème se fait avec un graphe dont les nœuds sont les membres de la communauté concernée, et dont les arêtes (non orientées) indiquent qui se connaît : on place une arête entre A et B si A et B se connaissent et donc s'influencent peut-être d'un jour à l'autre.

Le « théorème de la période 1 ou 2 » de Goles et Olivos stipule que, quelle que soit la répartition initiale des avis sur le graphe, l'application de la dynamique majoritaire conduit, en un nombre fini d'étapes, à une stabilisation complète des avis, ou à une double configuration des avis, chacune produisant l'autre.

Comme on le voit dans l'encadré ci-contre, si l'on part d'une configuration où le OUI l'emporte, le système se stabilise dans certains cas comme on l'attend en adoptant une configuration où le OUI reste majoritaire. Parfois cependant, la stabilisation se produit en adoptant une configuration où le NON l'emporte. Une troisième possibilité est prévue par le théorème de Goles et Olivos : le système se met à osciller entre deux états E et F. Il se peut même (*voir l'encadré 1, A3 et B3*) que pour E le OUI soit majoritaire et que pour F le NON soit majoritaire.

L'application de la dynamique majoritaire n'est donc pas le meilleur moyen d'arriver à un accord unanime, ou même seulement à un choix satisfaisant. Cependant, ce type d'évolution d'un graphe est très naturel et se rencontre dans plusieurs domaines scientifiques : réseaux d'automates finis, systèmes de votes, immunologie, interactions de cellules, réseaux de neurones, reconnaissance de formes, thermodynamique, etc. Le livre d'Eric Goles et Servet Martínez, *Neural and Automata Networks – Dynamical Behavior and Applications* >

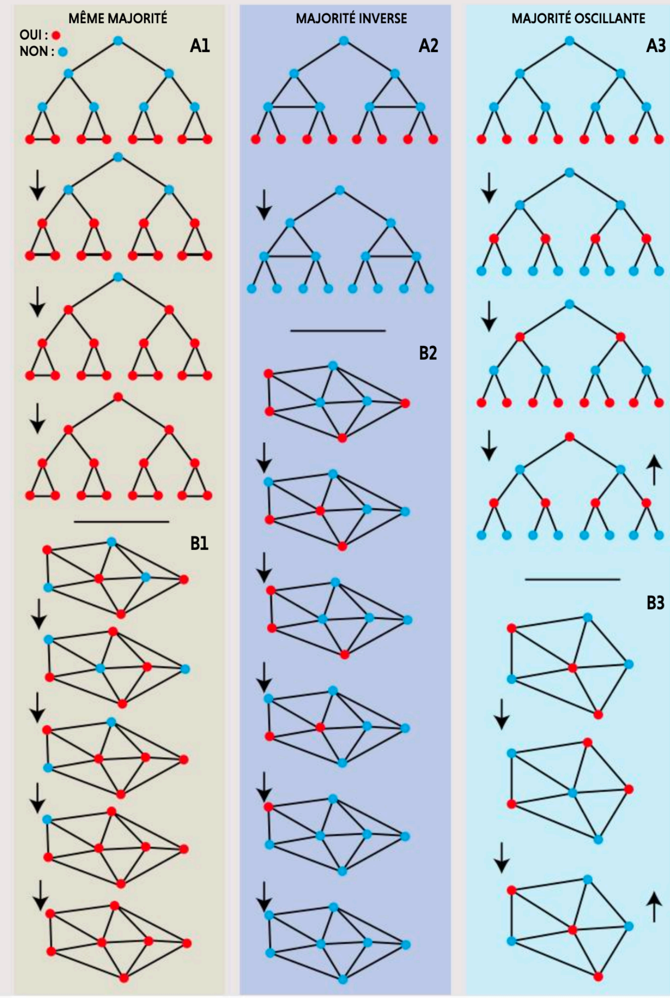
LE THÉORÈME DE LA PÉRIODE 1 OU 2

1

Adopter l'avis majoritaire (« oui » ou « non ») de ses voisins pour les nœuds d'un graphe définit un mode d'évolution des opinions. Selon le graphe et selon l'état des opinions de chaque nœud, cette évolution dure plus ou moins longtemps avant soit de se stabiliser, soit d'osciller entre deux configurations.

Les dessins A1 et B1 illustrent le cas où la dynamique majoritaire conduit à un état stable avec la même majorité qu'au départ. Dans les dessins A2 et B2, le graphe d'opinions se stabilise dans une configuration dont la majorité est inverse de celle de départ. Les dessins A3 et B3 montrent la troisième possibilité : le graphe finit par osciller entre deux configurations.

Le théorème de Goles et Olivos, ou théorème de la période 1 ou 2, indique que des cycles entre plus de deux états sont impossibles : une dynamique majoritaire sur un graphe conduit soit à une stabilisation, soit à un cycle binaire.



Pour la Science/Dec 2018/ 494

Genetic and regulatory networks
Neutral space and applications

Dante Travisany, E. G, Mauricio Latorre, María-Paz Cortes, Alejandro Maass, Generation and robustness of Boolean networks to model Clostridium difficile infection, Natural Computing, Springer, Nature, https://doi.org/10.1007/s11047-019-09730-0(0123456789().,-volV)(0123456789,-().volV) , 2019.

Ruz, G.A., Zúñiga, A., G. E. A Boolean network model of bacterial quorum-sensing systems, *International Journal of Data Mining and Bioinformatics*, Vol. 21, 2018, 123-144.

G, E., Ruz, G.A. Dynamics of neural networks over undirected graphs, *Neural Networks*, Vol. 63, 2015, 156-169.

Ruz, G.A., Timmermann, T., Barrera, J., G, E. Neutral space analysis for a Boolean network model of the fission yeast cell cycle network, *Biological Research*, Vol. 47, 2014, 64.

Ruz, G.A., G, E., Montalva, M., Fogel, G.B. Dynamical and Topological Robustness of the Mammalian Cell Cycle Network: A Reverse Engineering Approach, *Biosystems*, Vol. 115, 2014, 23-32.

G, E., Montalva, M., Ruz, G.A. Deconstruction and dynamical robustness of regulatory networks: application to the yeast cell cycle networks, *Bulletin of Mathematical Biology*, Vol. 75, 2013, 939-966.

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Ruz, G.A., G, E., Sené, S. Reconstruction of Boolean regulatory models of flower development exploiting an evolution strategy, The 2018 IEEE Congress on Evolutionary Computation (IEEE CEC 2018), Rio de Janeiro, Brazil, July 8-13, 2018, pp. 1-8.

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Ruz, G.A., Montalva, M., G, E. On the preservation of limit cycles in Boolean networks under different updating schemes, Advances in Artificial Life, ECAL 2013, 12th European Conference on Artificial Life, Taormina, Italy, September 2-6, pp.1085-1090, 2013.

Ruz, G.A., Timmermann, T., G, E. Building synthetic networks of the budding yeast cell-cycle using swarm intelligence. IEEE the Eleventh International Conference on Machine Learning and Applications (ICMLA 2012), Boca Raton, Florida, USA, December 12-15, pp. 120-125, 2012.

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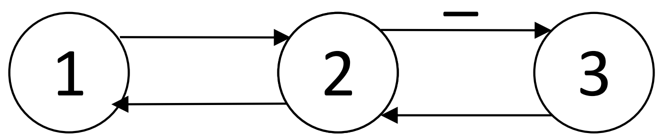
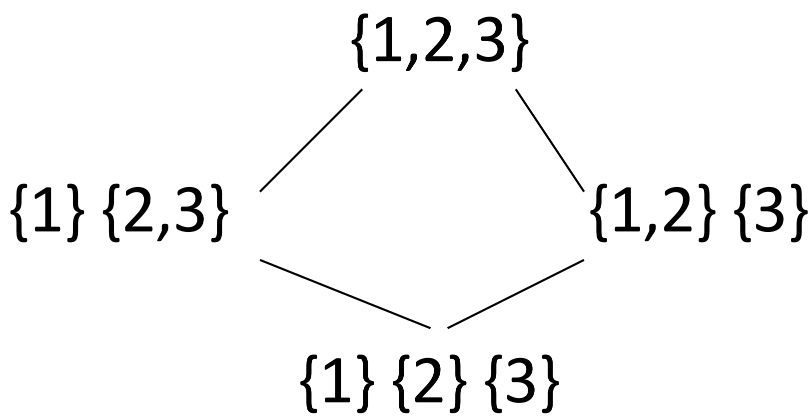


Block Sequential partitions for three elements

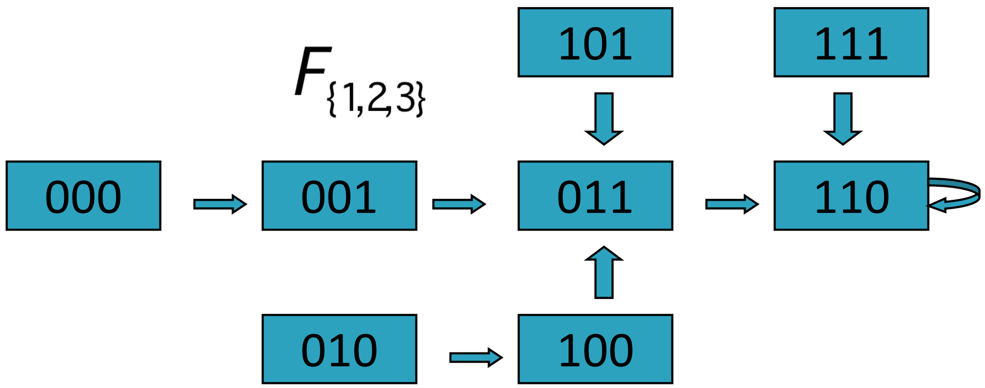


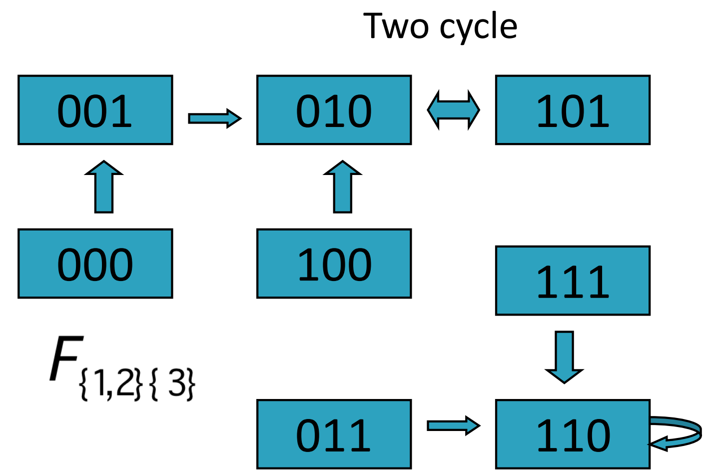
$$F_{\{1,2,3\}}(x_1, x_2, x_3) = (x_2, x_1 + x_3, \neg x_2)$$

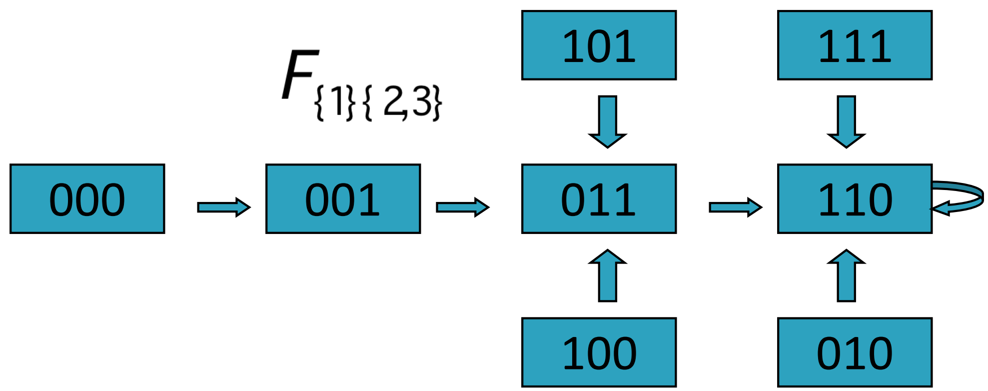
$$F_{\{1,2\}\{3\}}(x_1, x_2, x_3) = (x_2, x_1 + x_3, (\neg x_1)(\neg x_3))$$

$$F_{\{1\}\{2,3\}}(x_1, x_2, x_3) = (x_2, x_2 + x_3, \neg x_2)$$

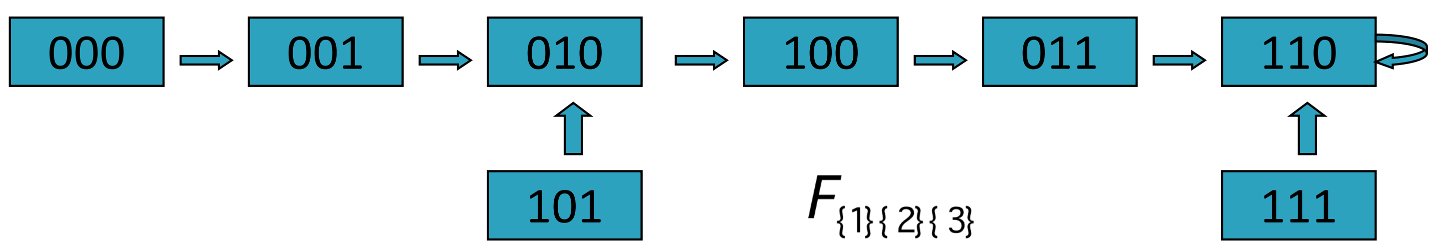
$$F_{\{1\}\{2\}\{3\}}(x_1, x_2, x_3) = (x_2, x_2 + x_3, (\neg x_2)(\neg x_3))$$





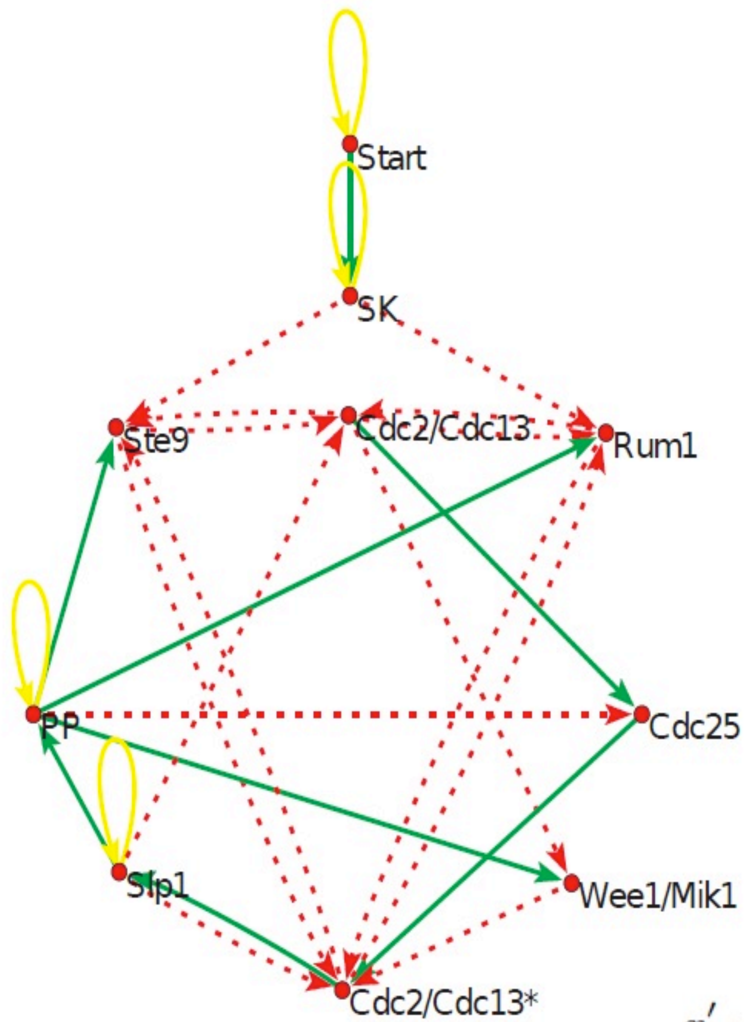


Block sequential diagrams



Fission yeast cell-cycle model (*Yeast1*)

Model proposed in Davidich, Bornholdt (2008) PlosONE)



$$x_i' = H\left(\sum_{j=1}^{n} w_{ij}x_j - \theta_i\right) = \begin{cases} 0 & if \quad \sum_{j=1}^{n} w_{ij}x_j - \theta_i < 0 \\ 1 & if \quad \sum_{j=1}^{n} w_{ij}x_j - \theta_i > 0 \\ x_i & if \quad \sum_{j=1}^{n} w_{ij}x_j - \theta_i = 0 \end{cases}$$

Fig. 3 The fission yeast cell-cycle threshold Boolean network. Using the same configuration as [3], (color online) the green/solid edges represent positive weights (activations), the red/dashed edges represent negative weights (inhibitory). The yellow/solid loops represent self-degradation.

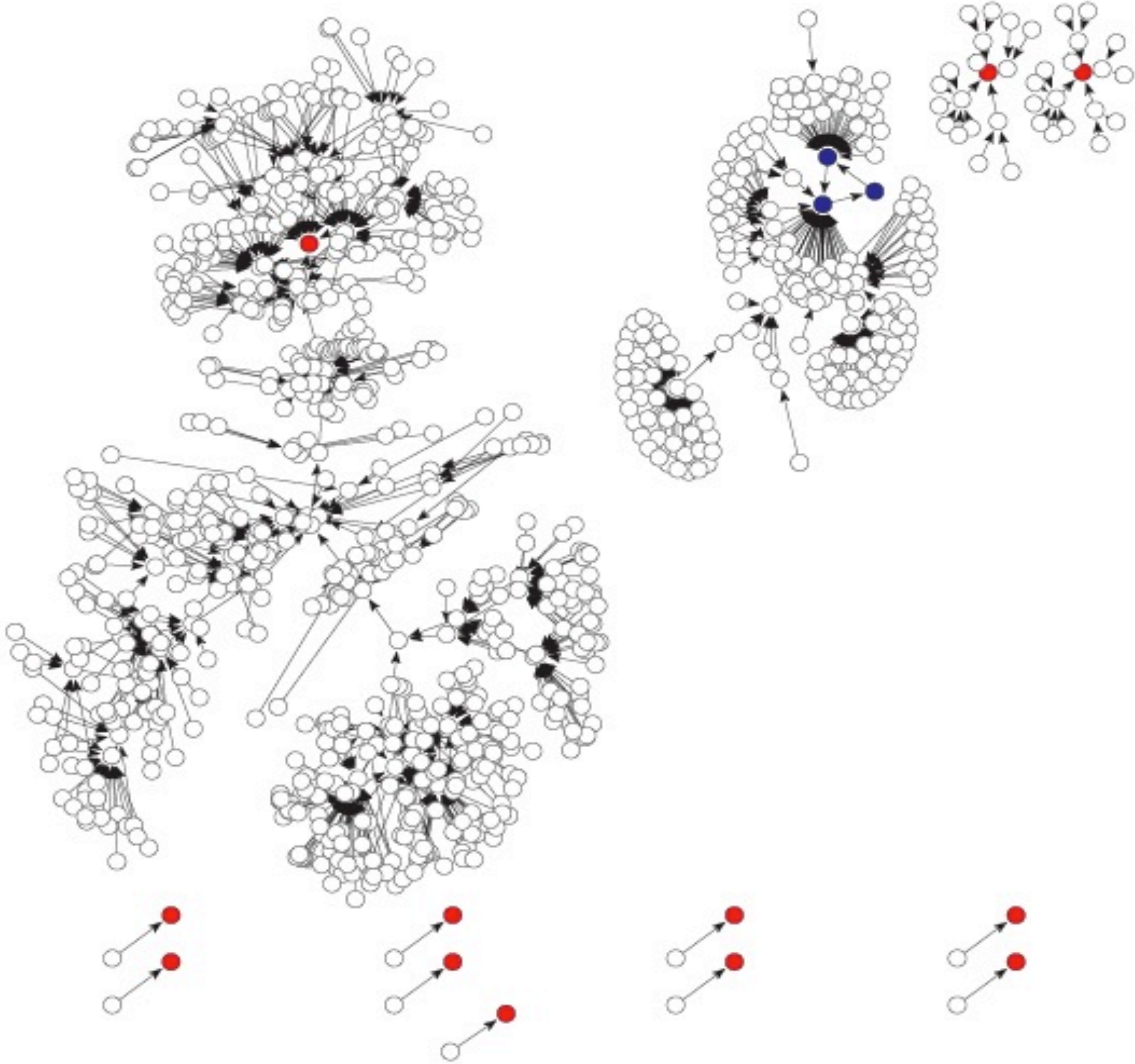


Fig. 7 State transition graph for *Yeast1* using the parallel updating scheme. (Color online) The twelve red circles represent the fixed point states, the three blue circles represent the states that belong to the limit cycle.

The total number of updates is 545835

The equivalence classes are 15350

For $s \in P(z)$ *and Ste9(t)=Rum(t)=*0 there are
3984 equivalence classes with a limit cycle (unique).

For $s \notin P(z)$ *there are 868 classes with a limit cycle (unique)*
Such that Ste9=0 and 661 classes with Ste9 non constant.

So there exists 5513 classes with a cycle
(period between 2 and 5)

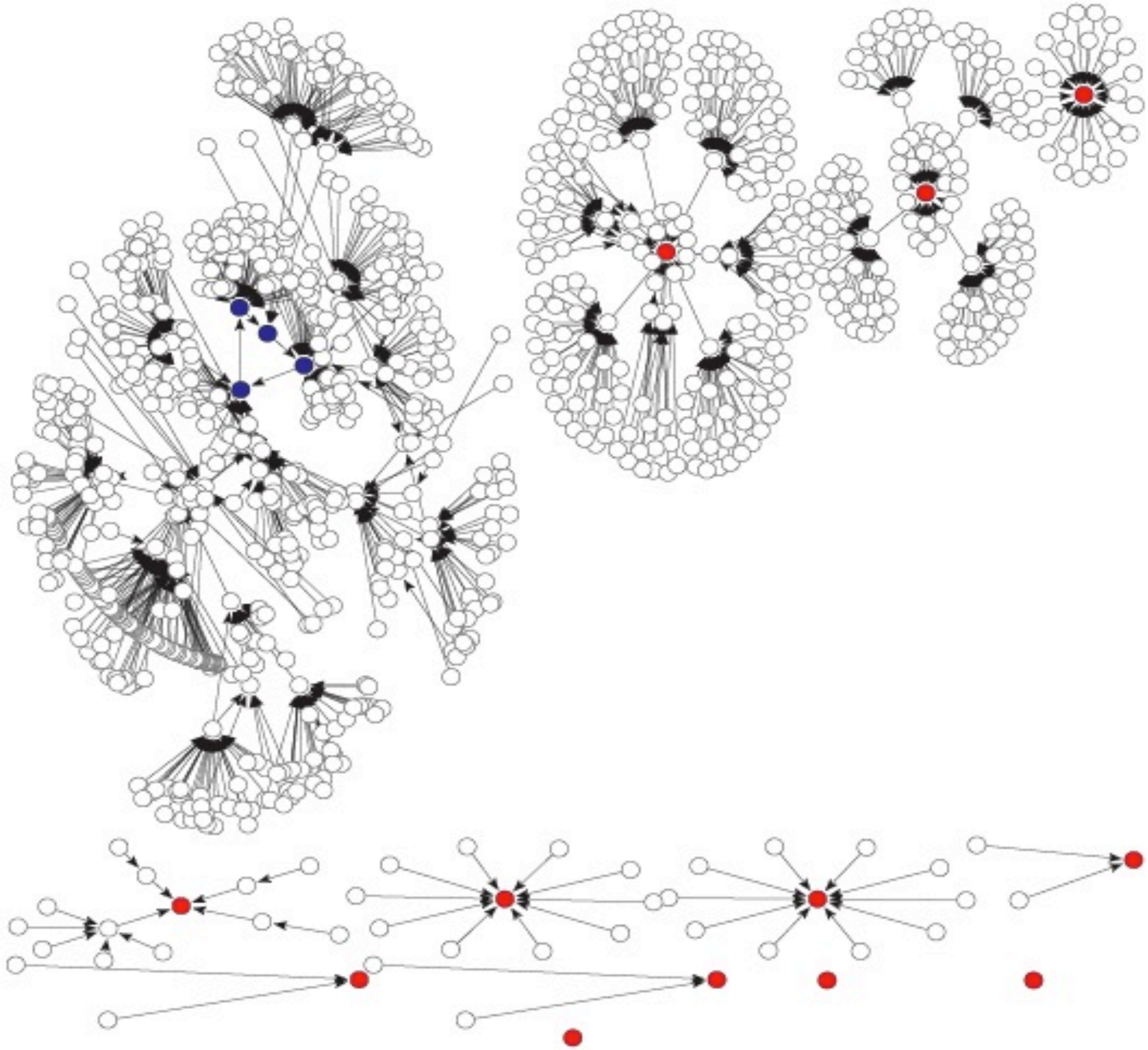


Fig. 8 State transition graph for *Yeast1* using the block-sequential updating mode: $s(Wee1/Mik1) = s(Cdc25) < s(Cdc2/Cdc13) = s(Cdc2/Cdc13^*) = s(Ste9) < s(Rum1) = s(Slp1) = s(PP) = s(Start) = s(SK)$. (Color online) The twelve red circles represent the fixed point states, the four blue circles represent the states that belong to the limit cycle.

Cell cycle of the budding yeast

Li, Long, Lu, Tang,(2004) The yeast cell-cycle network is robustly designed, PNAS,101, 4781-4786

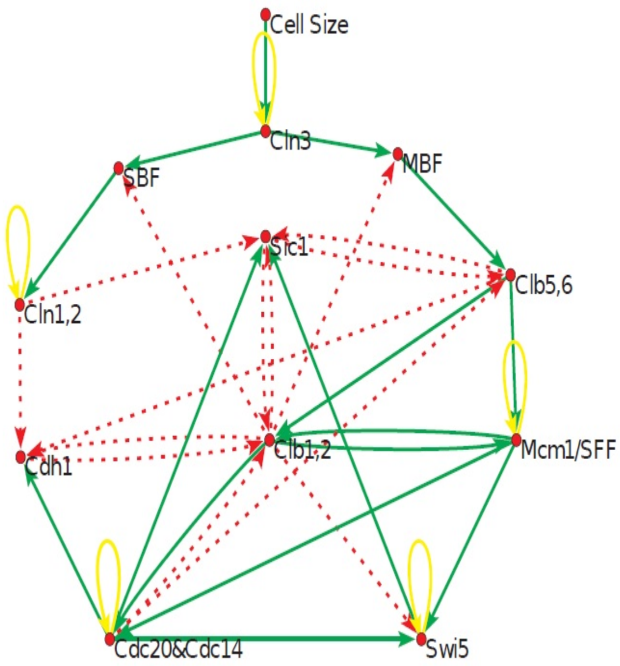


Fig. 12 The budding yeast cell-cycle threshold Boolean network. Using the same configuration as [14], (color online) the green/solid edges represent positive weights (activations), the red/dashed edges represent negative weights (inhibitory). The yellow/solid loops represent self-degradation.

THEOREM: for any update YEAST2 has only fixed points

ANTS as Complex systems: could be intelligence an emergent property?

A. Gajardo, E.G., A Moreira, Complexity of Langton's Ant, Discrete Applied Mathematics. Vol. 117, Issue 1-3, pp. 41-50, (with A. Gajardo, A. Moreira). (2002)

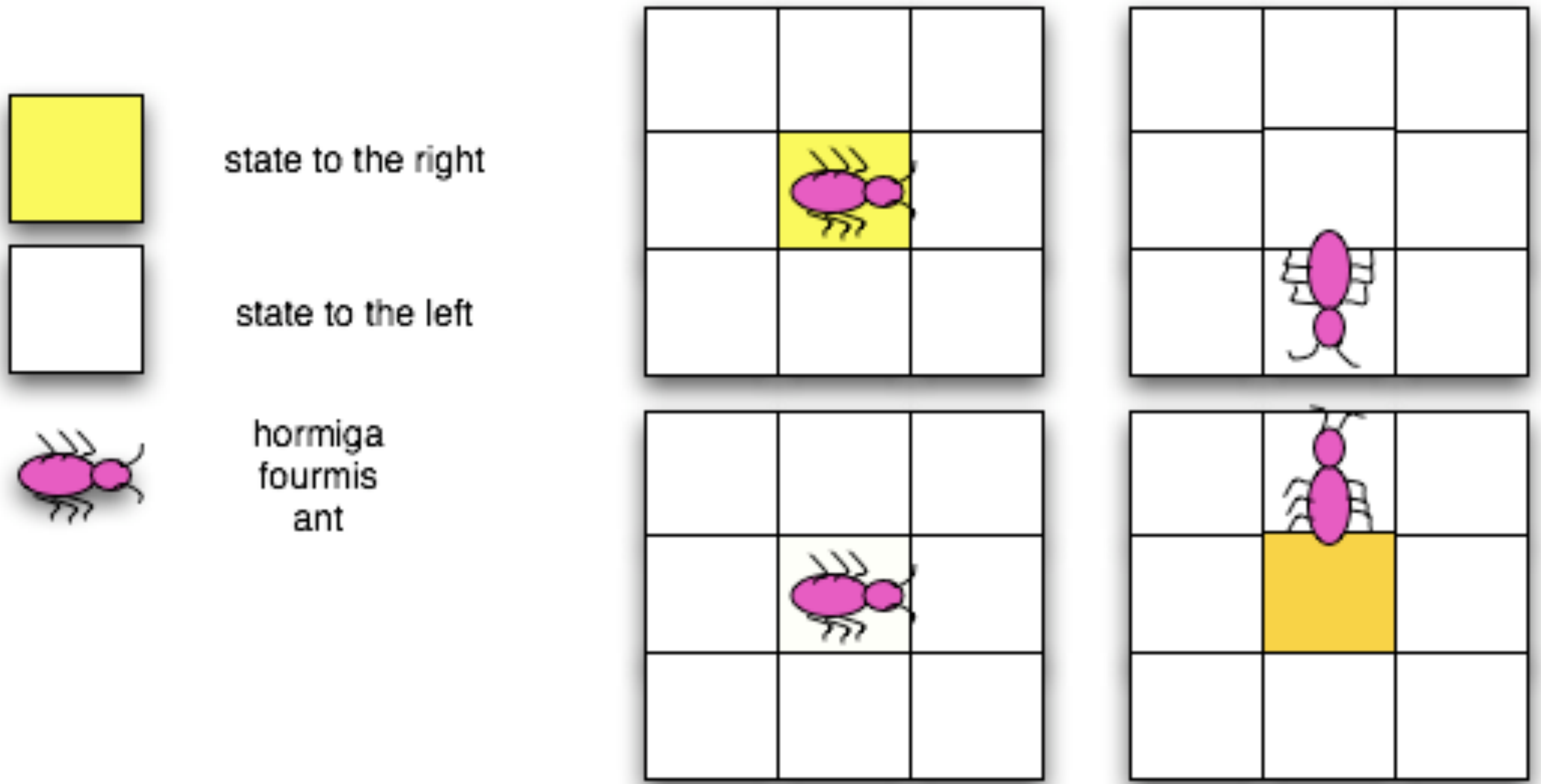
A. Gajardo, E.G, E. Moreira, Generalized Langton's Ant: Dynamical Behavior and Complexity, STACS'2001 Lectures Notes in Comp. Sci. 2010, pp.259-70. (2001)

A.Gajardo, E.G, A. Moreira, Dynamical Behavior and Complexity of Langton's Ant, in Complexity, Vol.6 , N°4, pp 46-51 (2001)

A. Gajardo, E.G., Dynamics of a class of ants in a one dimensional lattice in Theorical Computer Science, vol. 322, number 2, pp. 267-283 (2004).

M. Schimick, E.G, M. Markus, Tracks Emerging by Forcing Langton´s Ant with binary sequences, in Complexity 11,27-32 (2006).

Planar ant model (Langton's ant)



Ant's dynamics

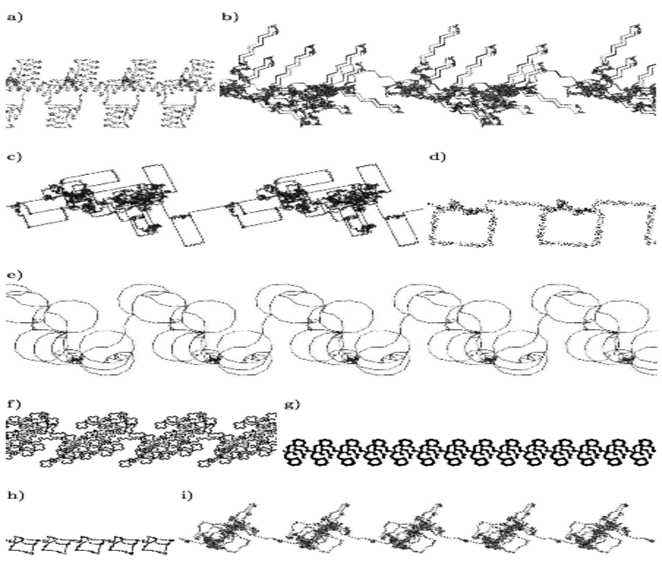


FIGURE 2: Same as Fig. 1, but with a one-bit perturbation in the forcing period.

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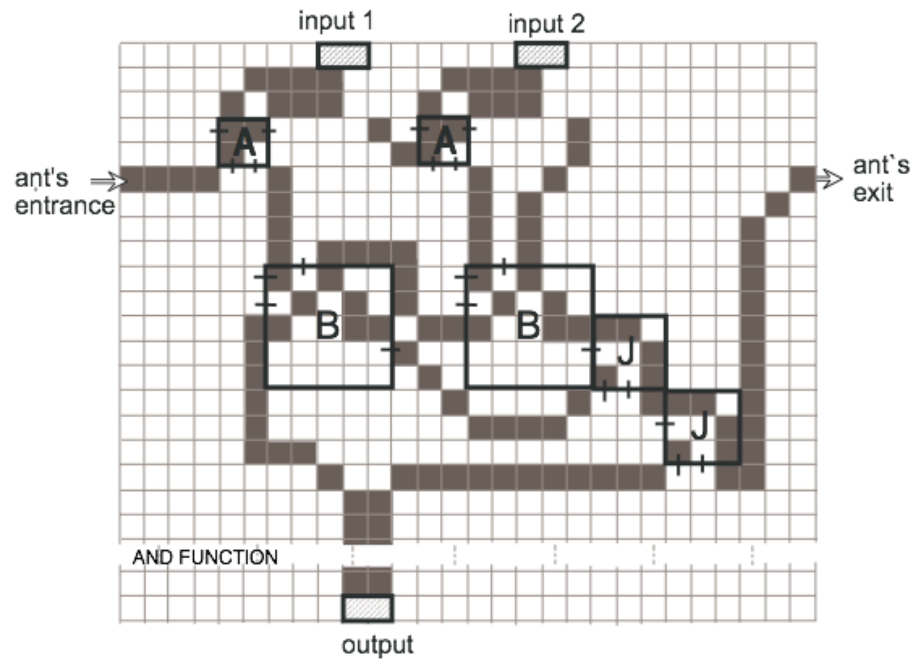
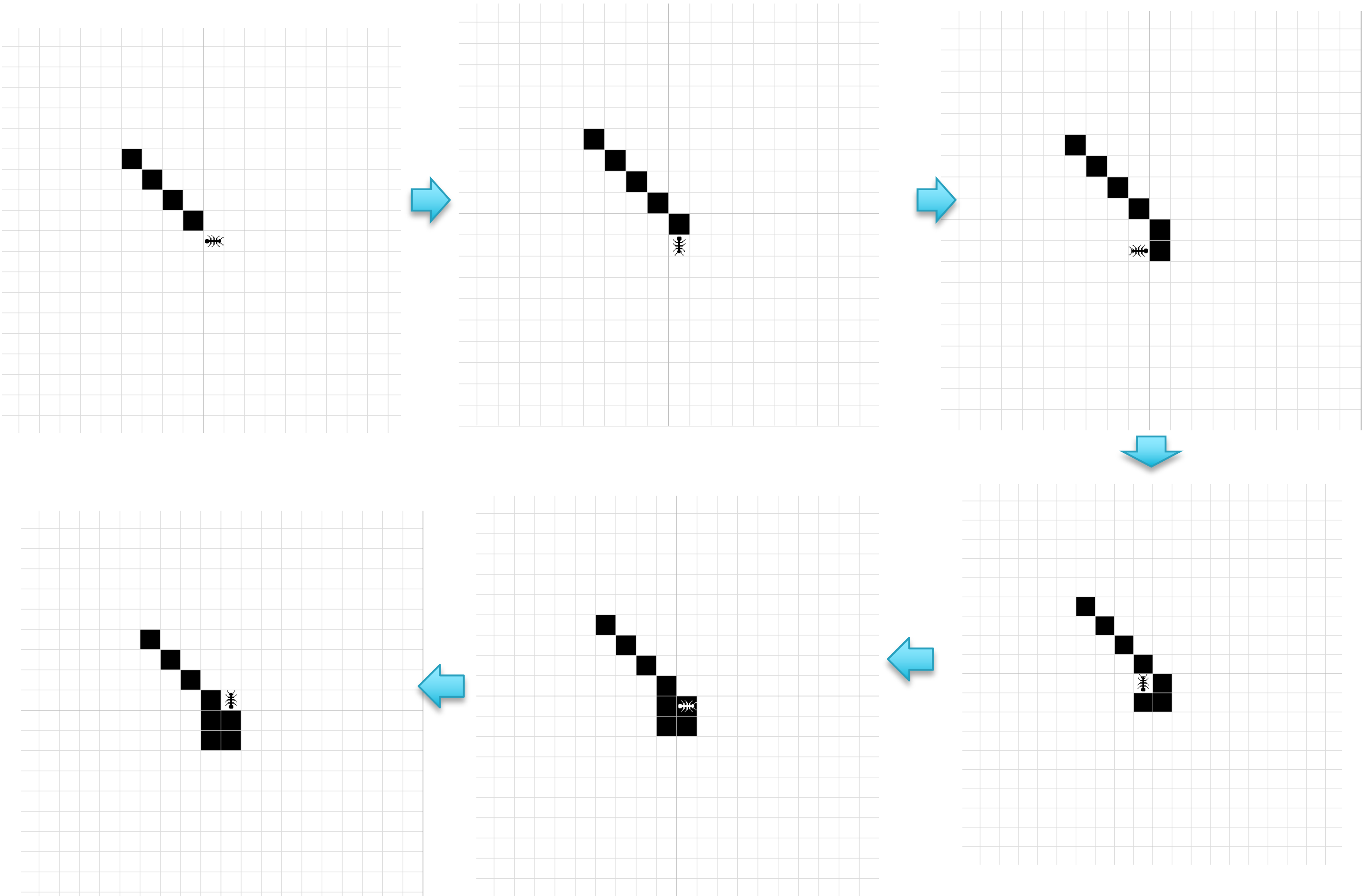
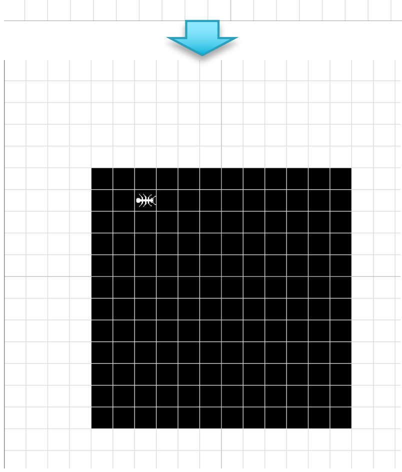
Logic gate



THEOREM: The ant is P-Complete and Turing Universal





Freezing ant.

Theorem: bounded behavior

Work in progress

EG, P. Montealegre, M. Ríos Wilson (Phd student) , D. Maldonado (post-Doc), P.Concha (Bío-Bío, DISC), R. Torres (Bío-Bío)

Sand Piles and Chip firing games

E. G, M. Kiwi, One dimensional sand piles, cellular automata and related models, in Procc. of Int. Conf. in Non-Linear Phenomena, Elsevier, P. Cordero et. al. eds., pp. 169-185, 1991.

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E.G., Sand pile automata, in Ann. Inst. Henri Poincaré, Vol. 56, Nº 1, pp. 75-90, 1992.

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E.G., M. Margenstern, Sand-pile as a universal computer, J. of Modern Phys. - C, 7(2), pp 113-122 (1996)

E.G, E. Prisner, Source reversal and chip firing, in Theor. Comp. Sc., 233, pp. 287-295 (2000).

E.G., M. Morvan. Ha Duong PhanSand Piles and order structure of integer partitions, in Discrete Applied Maths., vol 117, issues 1-3, pp 51-64 (2002)

E. G., Ha Duong Phan, M. Morvan, Lattice structure and convergence of a game of cards, in Annals of Combinatorics, volume 6,327-335 (2002)

Let $G = (V, E)$ denote an undirected graph, each vertex being finitely many connected, but G being possibly infinite. A distribution of chips is set on the set of vertices of G, say x_i chips, $x_i \geqslant 0$, in vertex i. The local rule of the game is the following: if the number of chips, x_i, is not less than the degree d_i of vertex i, the vertex gives one chip to each of its neighbors. The game can be played asynchronously, (i.e. vertices with enough chips are updated one by one) or in parallel. The last iteration mode can be defined as follows:

$$x_i(t+1) = x_i(t) - d_i \mathbf{1}(x_i(t) - d_i) + \sum_{j \in V_i} \mathbf{1}(x_j(t) - d_j),$$

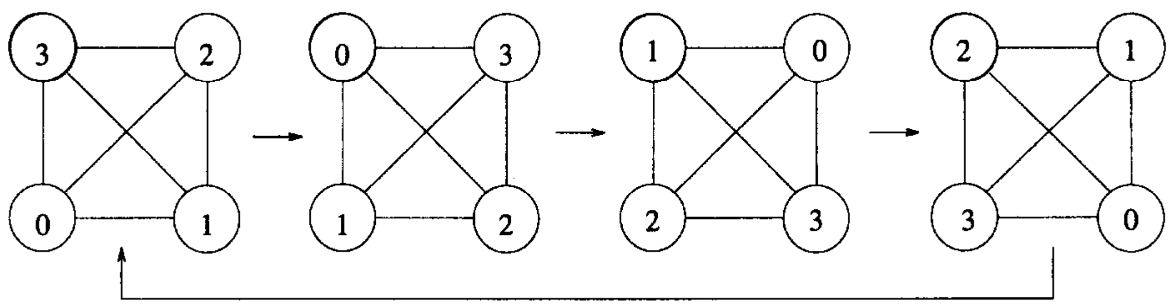

Fig. 1. Cycle of period four of the chip-firing game.

Wire

111102111
111110211

The OR gate:

1111111

2111111

1111111

The and gate

1111111

1111111

1111111

Chip firing game is P-complete, and Turing Universal

Recently we have established a relationship related with the complexity of C the chip firing and majority functions in a two dimensional grid with the von Neumann M neighborhood. (Work in progress, preprint, E.G., P. Montealegre, K, Perrot (2019)

Dynamics of automata networks and computational complexity

We will consider decision problems (YES or NO answer). The class P: problems which we can be decided in a serial computer in polynomial time.

The class NC: problems which can be decided in a parallel machine (say a PRAM) in poly-logarithmic time by using a polynomial number of processors.

Artículos (sólo UAI)

E.G., Pedro Montealegre-Barba, Ioan Todinca:*The complexity of the bootstraping percolation and other problems.* Theor. Comput. Sci. 504: 73-82 (2013)

E.G., Pedro Montealegre:*Computational complexity of threshold automata networks under different updating schemes.* Theor. Comput. Sci. 559: 3-19 (2014)

E.G, Pedro Montealegre:*The complexity of the majority rule on planar graphs.* Adv. Appl. Math. 64: 111-123 (2015)E.G, Pedro Montealegre:*A Fast-Parallel Algorithm for the Robust Prediction of the Two-Dimensional Strict Majority Automaton.* ACRI 2016: 166-175.

E.G, Pedro Montealegre, Ville Salo, Ilkka Törmä:*PSPACE-completeness of majority automata networks*. Theor. Comput. Sci. 609: 118-128 (2016)

E.G, Pedro Montealegre, Javier Vera:*Naming Game Automata Network*. J. Cellular Automata 11(5-6): 497-521 (2016

E.G, Diego Maldonado, Pedro Montealegre, Nicolas Ollinger:*On the Computational Complexity of the Freezing Non-strict Majority Automata.* AUTOMATA 2017: 109-119

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E.G, Pedro Montealegre, Kévin Perrot, Guillaume Theyssier:*On the complexity of two-dimensional signed majority cellular automata*. J. Comput. Syst. Sci. 91: 1-32 (2018)

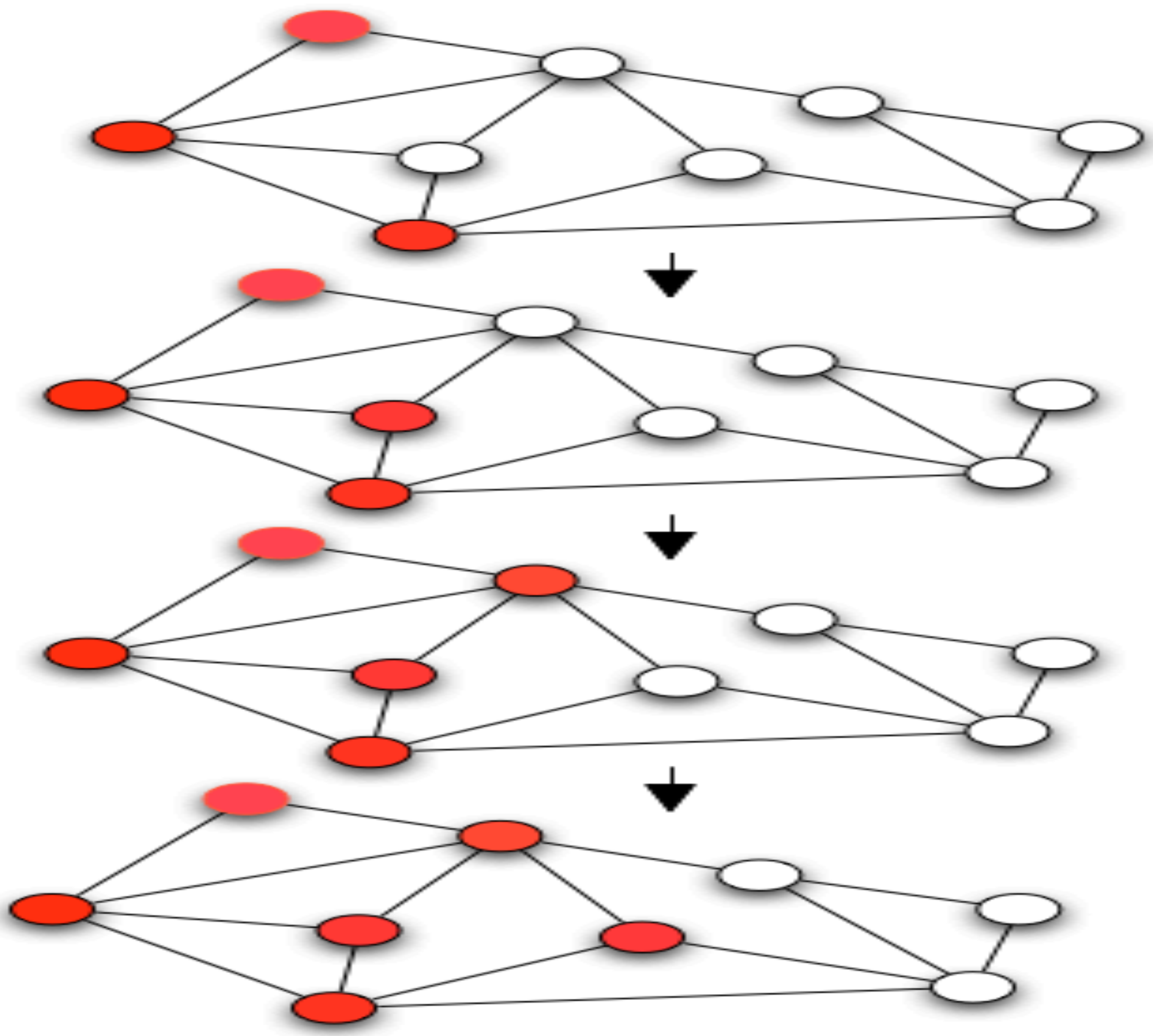
E.G, Pedro Montealegre, Martín Ríos Wilson:*On the Effects of Firing Memory in the Dynamics of Conjunctive Networks*. Automata 2019: 1-19E.G, Pedro Montealegre: *The complexity of the asynchronous prediction of the majority automata*. Information and Computation. (Accepted in 2019)

E.G, Diego Maldonado, Pedro Montealegre, Nicolas Ollinger:*On the Complexity of the Stability Problem of Binary Freezing Totalistic Cellular Automata*. Information and Computation. (Accepted in 2019)

Bootstrap Percolation

Given a finite non oriented graph G=(V,E) and an initial configuration of 0's and 1's.Consider the strict majority function operating at each node.

What is the relationship between the graph and the proportion of 1's such that iterated in parallel every node will become 1?



Decision problem PRE: given an initial configuration and a specific node at value 0. does there exist T>0 such that this node becomes 1?

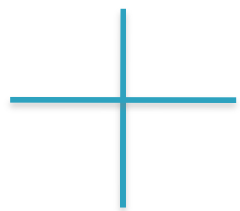
Theorem (E.G, P. Montealegre)

For graphs such that its maximun degree ≥ 5, PRE is P-complete.

If the maximun degree ≤ 4, PRE belongs to NC

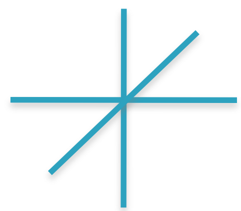
COMPLEXITY for the majority

We consider the similar decision problem PRE



Von Neumann neighborhood
in 2D
OPEN

conjecture (C. Moore): easy

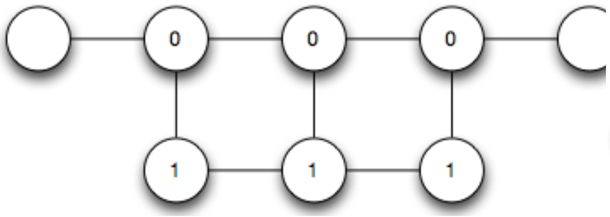


Nearest neighborhood
In 3D
P-Complete

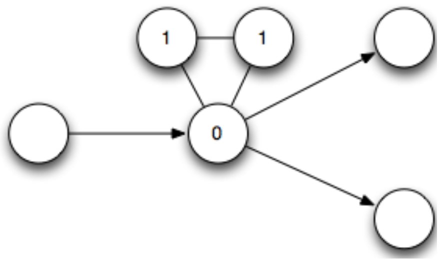
(C. Moore)

THEOREM: For planar graphs PRE for the majority vote is P-Complete

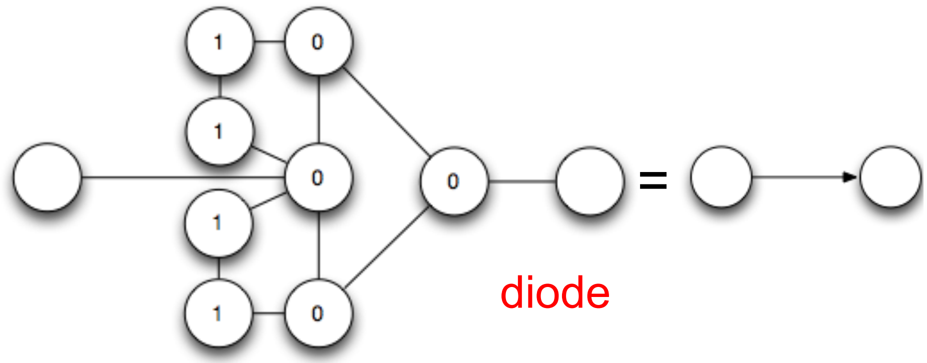
E.G, P. Montealegre.



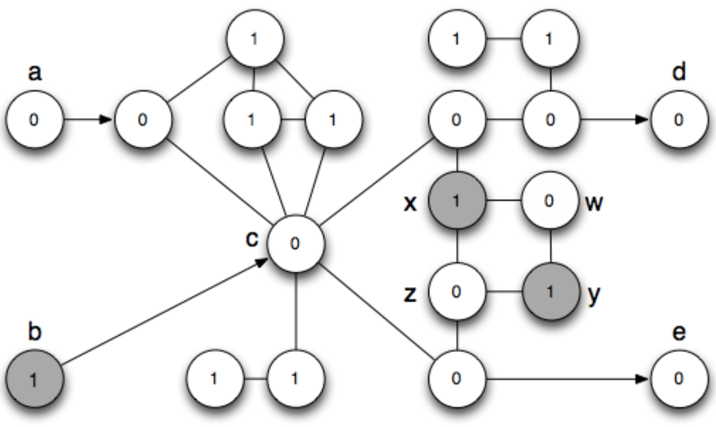
wire



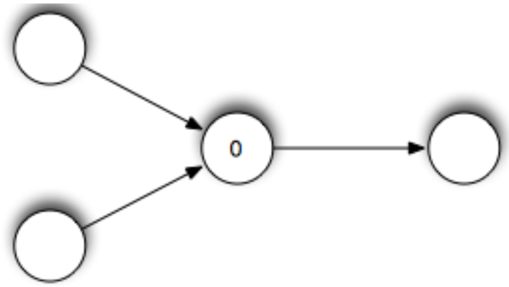
Duplicate a signal



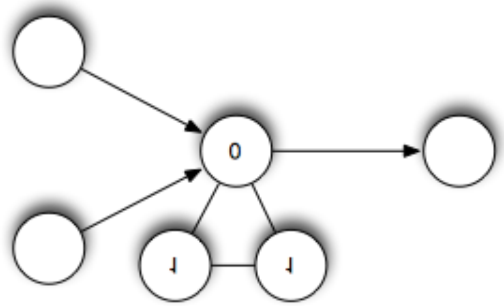
diode



The cross-over gadget



AND



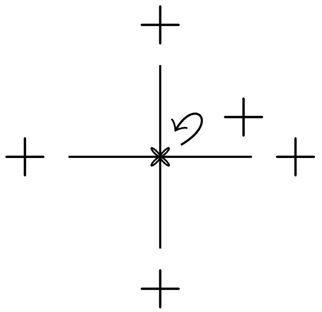
OR

Signed two dimesional Majority with the Von Neumann neighborhood.

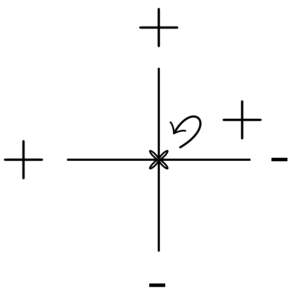
E.G., P. Montealegre, K. Perrot, G. Theyssier, On. The complexity of two diomensional Signed majority cellular automata, Journal of Computer and X System Sciences, Vol 91 Pp 1-32, 2018

Non-equivalent rules

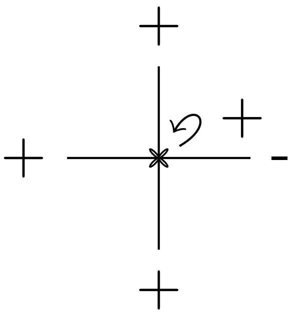
- Symmetric

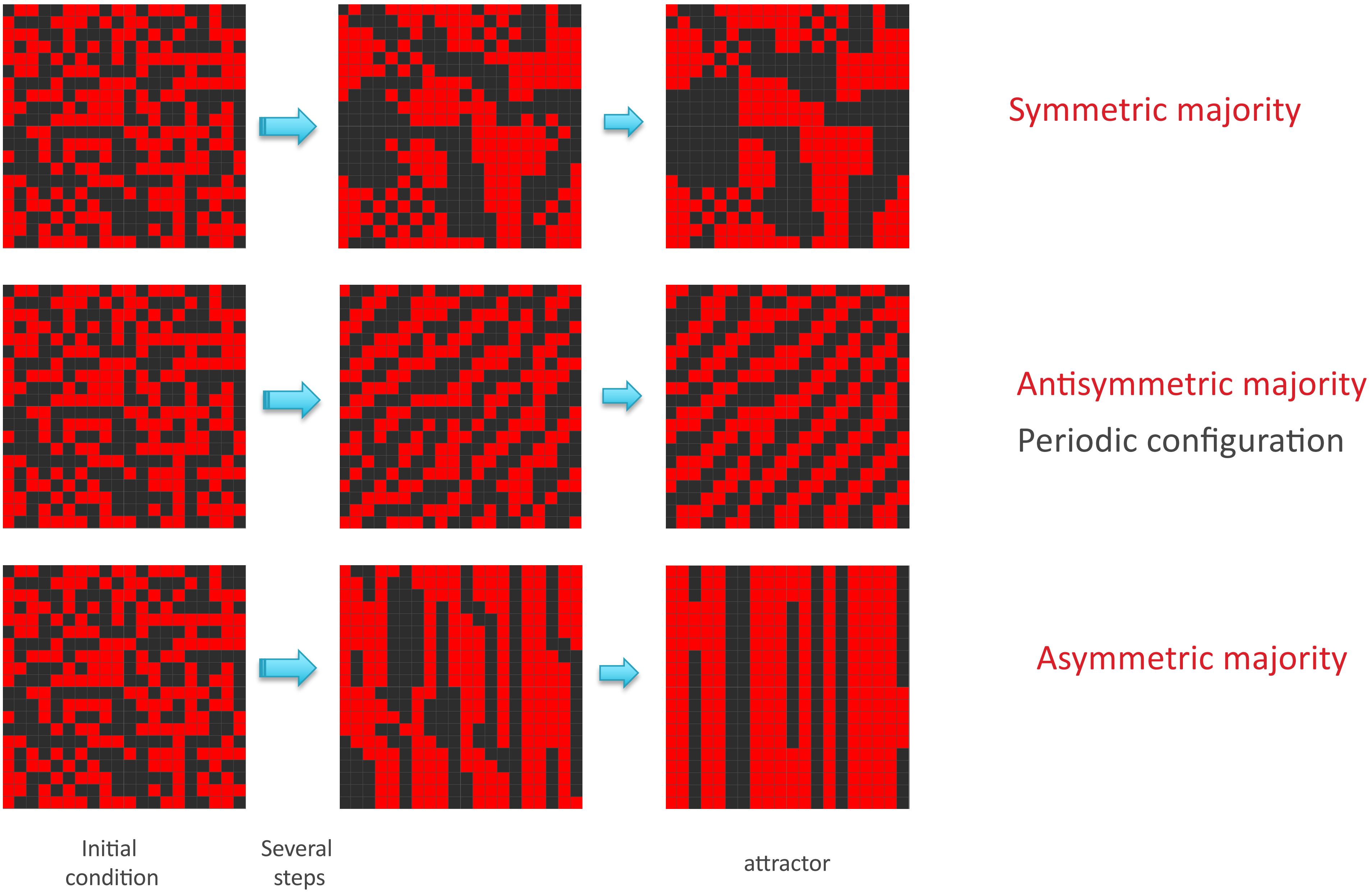


- Antisymmetric



- Asymmetric




Symmetric majority
Antisymmetric majority
Periodic configuration
Asymmetric majority
Initial condition
Several steps
attractor

Prediction problem

PREDICTION

Input:

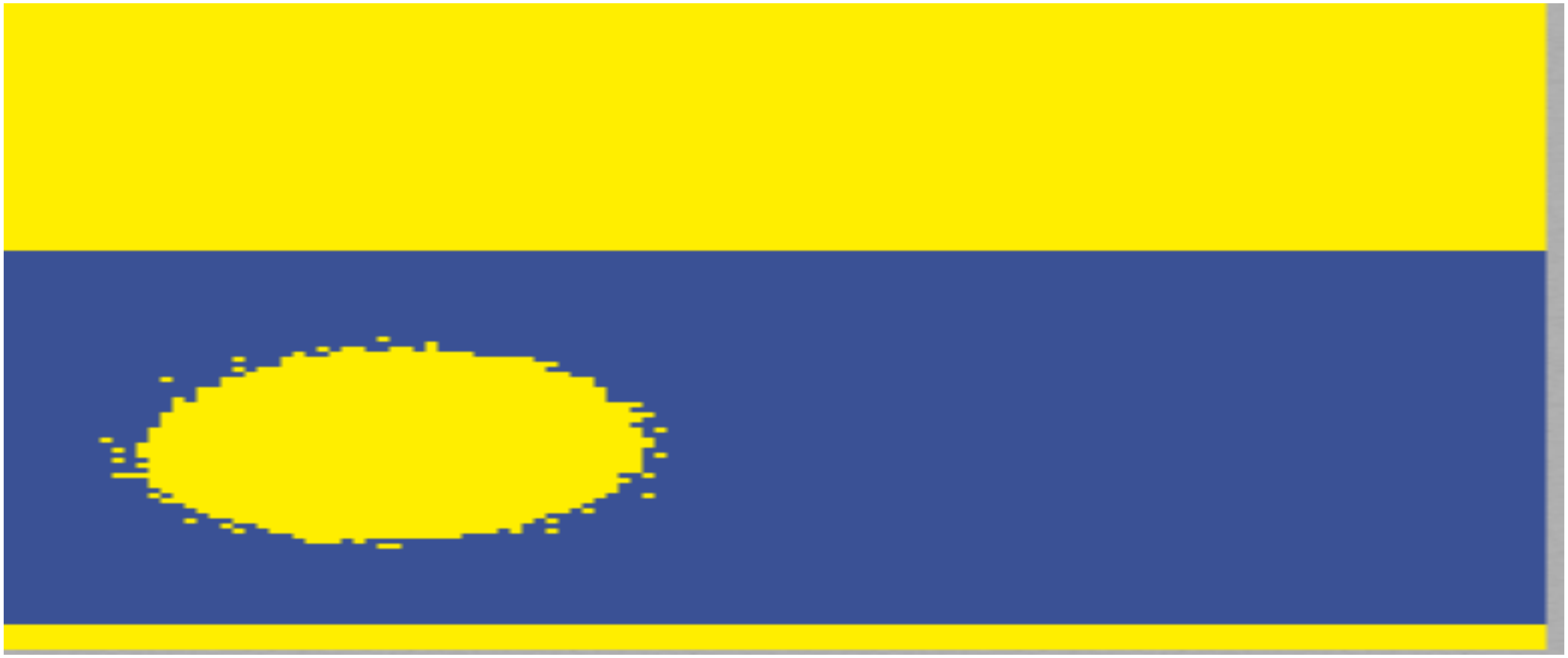
- a finite configuration of size n (periodic boundary conditions),
- $T > 0$ and,
- a site v.

Task: Compute the state of v after T steps.

- Symmetric
  - Cycles of length 2,
- Antisymmetric
  - Cycles of length $\Omega(n)$
- Asymmetric
  - Cycles of length $2^{\Omega(n^{1/3})}$

- Symmetric
  - Cycles of length 2,
  - PREDICTION is polynomial,
  - PREDICTION is NC^1-Hard.
- Antisymmetric
  - Cycles of length $\Omega(n)$
- Asymmetric
  - Cycles of length $2^{\Omega(\sqrt{n \log n})}$
  - PREDICTION is P-Hard.

Social Science Modelling: Schelling Segregation, Sakoda's model and polarization



Artículos (sólo UAI)

Nicolas Goles-Domic, Sergio Rica, E.G., PHYSICAL REVIEW E 83, 056111 (2011).

Canals, C., Goles, E., Mascareño, A., Rica, S., Ruz, G.A. School choice in a market environment: individual vs. social expectations, *Complexity*, Vol. 2018, Article ID 3793095, 11 pages, 2018.

Mascareño, A., Goles, E., Ruz, G.A. Crisis in Complex Social Systems: A Social Theory View Illustrated with the Chilean Case, *Complexity*, Vol. 21, No. S2, 2016, 13-23

V. Cortés, P. Medina, E. G., R. Zarama, S. RicAttractors, statistics and fluctuations of the dynamics of the Schelling's model for social segregation. Eur. Phys. J.B. 88:25 (2015) a

P. Medina, S. Rica, E.G, R. Zarama, Self-Organized Societies: On the Sakoda Model of Social Interactions , Complexity, Volume 2017 (2017), Article ID 3548591, 16 pages, https://doi.org/10.1155/2017/3548591.

Böttcher, L., Woolley-Meza, O., G, E., Helbing, D., & Herrmann, H. J. Connectivity disruption sparks explosive epidemic spreading. Physical Review E, 93(4), 042315(2016)

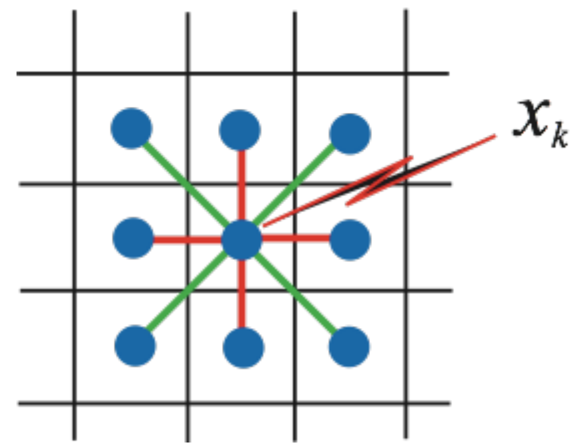
Lucas Bottcher,1, 2, ∗ Hans Gersbach,2 . E. G.,3 and Pedro Montealegre3, Competing Elites and Political Polarization 1Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland 2Center of Economic Research, ETH Zurich, 8092 Zurich, Switzerland 3Facultad de Ingenierı́a y Ciencias, Universidad Adolfo Ibáñez, Santiago, Chile (Dated: June 12, 2019)

The Model of Segregation by Shelling

Thomas C. Schelling (1969 - 1972)

Lattice one or two dimensional with periodic conditions

State $x_k = \pm 1$



Neighborhood Moore (green and red arrows) and von Neumann (red arrows)

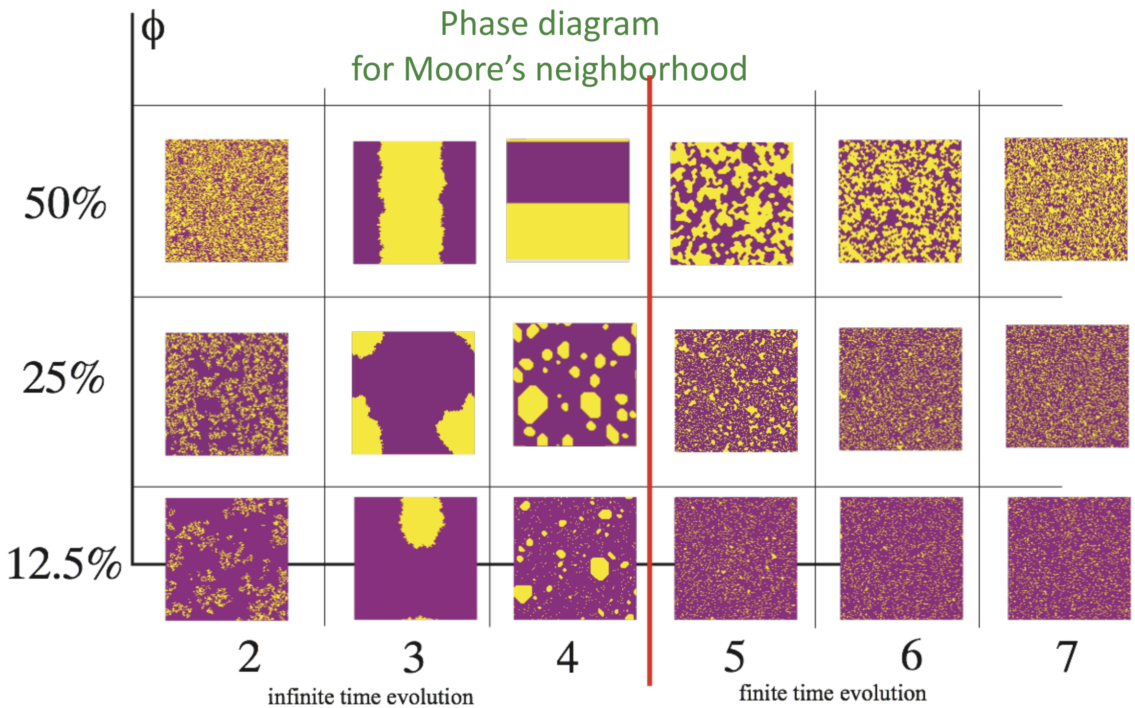
Tolerance threshold $\theta \in \{1, \ldots |V|\}$

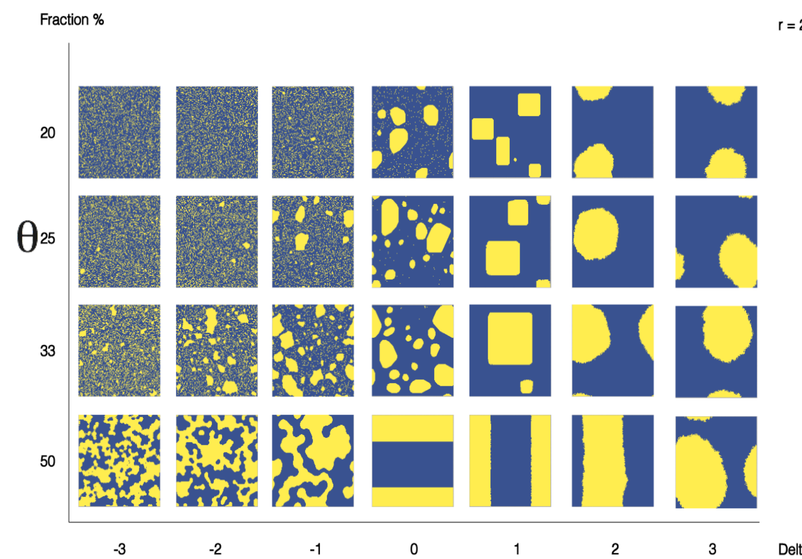
Happiness threshold

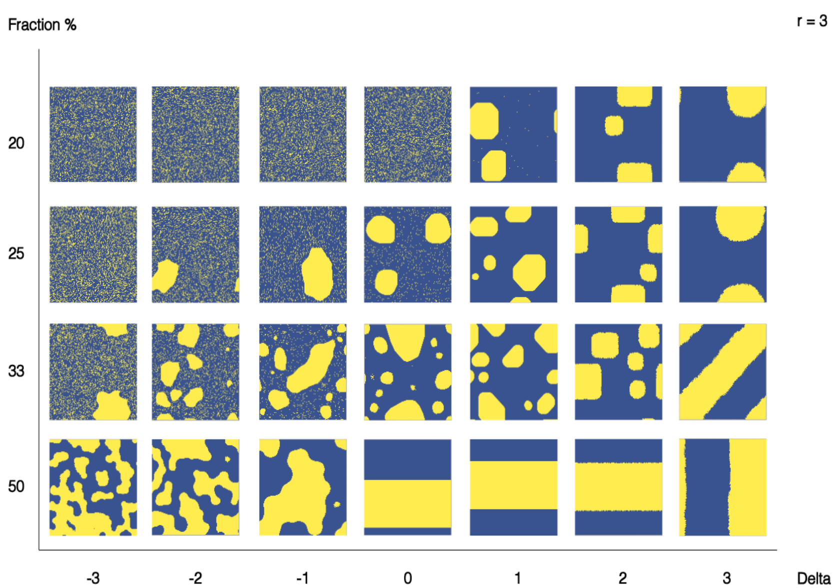
An individual is unhappy if there are more than θ individuals on the other state in its neighborhood

The update rule

At each step, one lists the unhappy individuals of both species, and then randomly (for instance) one exchanges two individuals of opposite value.



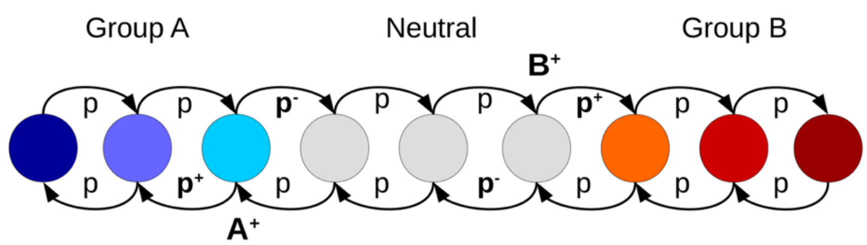




Other neighborhoods

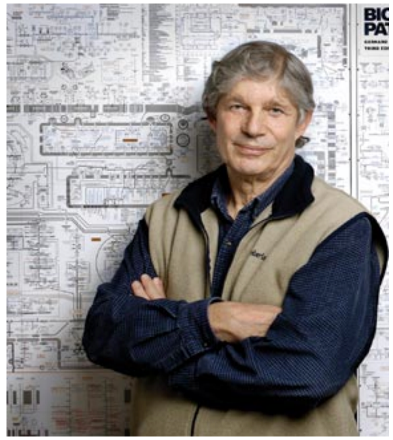
Polarization.

Recent empirical findings suggest that societies have become more polarized in various countries, i.e. the median voter of today represents a smaller fraction of society compared to two decades ago. What is driving this polarization? Activist-voter interactions play a major role in political opinion formation. We study a macroscopic opinion model in which activists target certain groups of individuals in order to inject their political ideas. Polarization emerges when small heterogeneities among competing activists cause them to target different groups in society



Polarization model. In this example, the political spectrum consists of N = 9 different states and is divided in three groups: group A, a neutral set of agents, and group B. A transition from one state to its nearest neighbors occurs with probability p. A political activist A_+ or B_+ can locally decrease the transition probabilities ($p_- < p$) or increase them ($p_+ > p$).

Lucas B¨ottcher, Hans Gersbach, (ETH) E.G. P. Montealegre.UAI

Tchuente



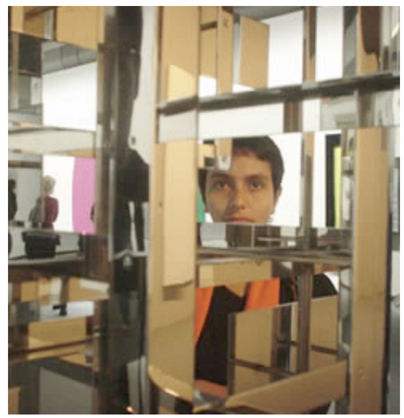
































See you soon !!